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EXTENSION OF APOLLO MIDCOURSE  
NAVIGATION AND GUIDANCE TO THE MARS MISSION

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EXTENSION OF APOLLO MIDCOURSE NAVIGATION AND GUIDANCE  
TO THE MARS MISSION

SUMMARY

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This paper presents results from a preliminary assessment of the application of Apollo navigation and guidance techniques to the Mars mission. A 5-second RMS sextant was found to be capable of measuring the midcourse position and velocity accurately to values about 300 nautical miles and less than 1 ft/sec, respectively. A schedule of one navigation sighting every 15 hours was proved to be sufficient to maintain the position and velocity estimate at the minimum level. In this preliminary assessment, midcourse velocity requirements of 400 ft/sec for the trans-mars phase and 600 ft/sec for the trans-Earth phase are indicated to be sufficient for injection and midcourse-correction velocity errors of 14 ft/sec or less.

AUTHOR

INTRODUCTION

The midcourse guidance and navigation problem for the Apollo mission is, in many ways, the most difficult problem ever undertaken in the controls field. The difficulty stems from the combination of a complex environment, heavy penalties for expenditure of energy, and the requirement for very high reliability for the manned mission. The next step, the interplanetary mission, does not represent nearly as large an extension of the art as was required for Apollo. The intent of this paper is to illustrate that direct application of Apollo techniques is at least sufficient to solve the problem.

APOLLO TO MARS MISSION  
SUMMARY

The contribution of the complex environment to the difficulty of the Apollo problem results from both the nonlinear nature of the mechanics of space flight and the uncertainties in the best geometrical and gravitational description of the problem available. This latter difficulty makes an on-board system mandatory, for the problem must be solved closed-loop. The function of the onboard navigation and guidance system is to determine the state vector (position and velocity) of the spacecraft (navigation) and to determine what changes in the velocity vector are required to arrive at the desired end conditions at some later time. Table I shows the major functions of the guidance system and the type of equipment needed to perform these functions. Navigation measurements are made by either a sextant which measures angles or a radar unit (electronic or LASER) which measures range and range rate. These measurements are used by the computer to determine the position and velocity forward in time to determine position and velocity error at time of arrival. By using an arrival-error prediction, calculations of corrective velocity are made and executed with the attitude reference and propulsion unit.

Clearly, the navigation and guidance system functions in the above manner for either a lunar mission or a mission to Mars. The techniques being developed for the Apollo mission are general techniques which can be applied to Mars mission or for that matter, missions to any other point in the solar system.

One basic assumption which is made here and will be carried through-out is that the navigation and guidance problems can be divorced; that is, once the navigation system provides the best estimate of the state, the guidance technique treats this information as if it were deterministic. This approach is adequate for the Mars as well as the Apollo mission, and it avoids the complexity of combining the problems in a program which is already complicated.

In this paper, the navigation technique and associated results are discussed, the guidance problem and an estimate of the fuel requirements are presented, and an assessment of the overall sufficiency of the Apollo techniques for the Mars mission is made.

#### NAVIGATION

The Apollo navigation technique is based on a dynamic filtering method originally developed by Kalman. Battin of MIT developed the filter independently for the particular case of scalar information (see ref. 1), and he and his associates added some very significant modifications for the problem at hand to eliminate the need for a reference trajectory and to insure stability despite numerical processing limitations.

The primary navigation information for the Mars mission is expected to be sextant measurements of the angle between a star and the Earth, Sun, or Mars. The measurement between the sun and a star is diagrammed

in figure 1. Without a detailed explanation, the action of the filter is illustrated in figure 2. Here the cone of position is the locus of all possible positions for a star-body measurement. The large, lightly shaded ellipse about the estimated position describes the uncertainty prior to the measurement, and the dark ellipse about the improved position estimate describes the uncertainty after the incorporation of the measurement.

A simulation of the Apollo navigation technique was used for a preliminary estimate of the accuracy of midcourse navigation for the Mars mission. Three cases, representing Earth-Mars midcourse, Mars approach, and Earth approach, were investigated using only angle measurements to determine the state of the spacecraft. The navigation system was run using a Monte-Carlo process to add errors to the sextant measurements. These errors were assumed to be normally distributed about a zero mean with a standard deviation of 5 seconds. The results of a series of runs were automatically compiled to determine the average error and standard deviation of the error in state. It was found that the results became essentially stationary after 25 runs.

Figure 3 shows the velocity and position errors during the midcourse phase of a 120-day trip to Mars in which angular measurements between a star and the Earth, Sun, and Mars were used to determine the state vector.

The initial errors for this run were 7 nautical miles in position and 15 ft/sec in velocity. The sighting schedule was 11 measurements each, Earth-Sun-Earth at 1-hour intervals inside the Earth's sphere of influence, then 10 measurements at 15-hour intervals in the following sequence: S E M S E S E M S E S E S M E S M S M S, where S is sun, E is Earth, and M is Mars. The stars used in making the angular measurements were chosen in a computationally optimum manner in which 20 bright, easily identified stars distributed as evenly as possible about the celestial sphere were used. The criterion for optimum star is that the cosine of the angular difference between the measured and computed angle must be a maximum. This technique was intended simply to minimize computation noise in handling the measurements.

The results show midcourse position and velocity errors in the vicinity of 250 nautical miles and 1.0 ft/sec until 1,000 hours, at which time the position errors jump to 1,000 nautical miles and 3.0 ft/sec. This large jump in position and velocity errors corresponds to the first Mars angular measurements and indicates that the Mars measurements were introduced too early in the trajectory. The position error is reduced, however, to about 300 nautical miles and 0.5 ft/sec at 1,600 hours and stays at this level for the remainder of the flight. The velocity error of 1 ft/sec to 3 ft/sec is well within the requirements for midcourse corrections. The position error of 300 nautical miles would be insignificant in the midcourse velocity correction. Indications are that one measurement every 15 hours is sufficient to maintain a good estimate of

the state vectors. Sightings at more frequent intervals are not expected to decrease the errors and one measurement in 24 hours would probably give nearly the same result.

Figures 4 and 5 show the performance of the navigation system for a sextant of 5-second accuracy on approach to the planets Mars and Earth, respectively. The schedule of angular measurements was 10 hours on the planet, 10 on the Sun and 10 on the planet just before periapsis. Measurements were made at  $\frac{1}{4}$ -hour intervals on approach to Mars and  $\frac{1}{2}$ -hour intervals on approach to the Earth. Initial errors of 325 nautical miles and 0.3 ft/sec at 29 hours were used for the Mars approach and 523 nautical miles and  $\frac{1}{2}$  ft/sec at 32 hours for the Earth approach.

By comparing the Mars approach (fig. 4) with the midcourse results shown in figure 3, it is observed that increasing the number of sightings from 1 every 15 hours to 4 per hour had very little effect on the position and velocity errors until the spacecraft approached to within 12 hours of periapsis. At 12 hours the position errors drop very rapidly while the velocity errors start to grow; velocity errors reach the highest value of 0.5 ft/sec to 1.0 ft/sec just before periapsis. The Earth approach (fig. 5) shows the same characteristics. The peaks in the velocity error just before periapsis are the result of a rapid growth in velocity error between navigation measurements. More frequent measurements in this region would improve the velocity estimate. Indications are, however, that the last velocity correction should be

made as quickly as possible after the last navigation sighting if entry errors are to be minimized.

The inability to reduce the position and velocity errors when the number of navigation sightings is increased at the sphere of influence is a result of the ineffectiveness of angular measurements in determining the position for hyperbolic approach. This effect is illustrated in figure 6. For large radii, the difference between  $\alpha_1$  and  $\alpha_2$  on a hyperbolic trajectory is very small and the approach disappears in the noise level of the measurement as the measurements are made more frequently. As the spacecraft approaches periapsis, the rate of change of  $\alpha$  gets larger, then the navigation measurements become more effective. Two deficiencies in the investigation so far are the arbitrary selection of measurement sources and the arbitrary sighting schedule employed. Clearly, alternate measurements, such as the Earth's moon or the moons of Mars, would provide more precise sources in certain areas. Moreover, the scheduling of sightings is susceptible to optimization, as shown in reference 2. It is expected to attack these areas in future work.

#### GUIDANCE

The midcourse guidance problem is defined as follows: given the current state of the vehicle, the desired elements of the state of the end condition, and a reasonable number of constraints, find the changes in state required to meet the end condition. In current "high-thrust" space guidance, the only change in current state which is practical



from a control viewpoint is a correction to the velocity vector.

Two general approaches to this problem exist. The implicit guidance approach converts the boundary-value problem to a regulator problem by defining a trajectory with the proper end conditions and then forcing the vehicle to move toward this trajectory in at least an asymptotically stable manner. The explicit guidance approach solves the boundary-value problem directly, by using only the location of the "target" as a reference. There is, of course, a middle ground of semi-implicit or semi-explicit approaches. For manned space missions, the possibility of abort and of changing objectives as data from the neighborhood of the target is acquired make an explicit guidance technique highly desirable.

For the Apollo mission, the relatively restricted time available for computing may prevent the mechanization of a strictly explicit guidance scheme. The technique currently envisioned is explicit to the sphere of influence of each reference planet; that is, targets either within or at the sphere of influence are used, and hence a dummy aim point at the sphere is necessary when the actual target is across the sphere. This technique at least minimizes the required references. For the Mars mission, considerably more computing time should be available, and a full-blown guidance mechanization of the  $n$ -body problem should be practical.

The iteration technique presently being considered for the Apollo mission is also applicable to the Mars mission. This technique is

essentially a delta correction technique which uses matched conic trajectories to evaluate the correction velocity. The matching is done at the sphere of influence of the planet. The procedures are as follows: first the measured position and velocity are integrated by the Encke method to obtain a precision estimate of the spacecraft flight path and the position at the predicted time of entry into the sphere of influence of the planet. This trajectory is indicated by the dashed line (labeled predicted path) in figure 1. A conic trajectory is calculated between the location of the spacecraft at time correction is to be made and the location at the time the spacecraft is to pierce the target planet's sphere of influence. A second two-body trajectory is calculated between the location at the time the correction is to be made and the desired end conditions or pierce point. The vector difference in these quantities is the guidance-correction velocity. Since an analytical solution exists for the two-body trajectories, these trajectories can be computed very rapidly. The most time consuming part of the computation is the integration of the trajectory to predict the spacecraft's path. This process is iterated by using the velocity resulting from the previous cycle until the desired accuracy is obtained. Experience gained by MSC in calculating lunar trajectories with matched conic trajectories indicates that this is a very stable technique. The error can be reduced by a factor of 10 or greater with each guidance computation.

The mechanization of a simulation of this guidance technique is probably not warranted until a specific qualitative description of Mars

mission trajectories is delineated, for instance, until the "hinging" of trajectories requiring plane changes is defined. At the present time it should be sufficient to note that since the scheme operates for the Earth-Moon problem, in principle, it should be easier to apply it to the Mars mission problem.

In order to develop a rough grasp of the guidance problem, a preliminary analysis was made of the guidance velocity requirements by using two-body motion of the spacecraft about the Sun and coplanar flight with the planet to be intercepted. This seemed a reasonable first approach since more than 97 percent of the flight time on a Mars mission would be outside the spheres of influence of Earth and Mars. An evaluation of the navigation data reveals that the velocity error never goes to zero even though the components may have zero means. These results imply the need for a high degree of correlation between the components of the velocity errors. Since the velocity error does not go to zero, a fairly large position error will exist at each midcourse correction, so that the guidance velocity requirements obtained by adding together requirements on the basis of consistent errors at each correction is a good assessment of the requirements for velocity errors of 14 ft/sec RMS.

Figures 7 and 8 show results for intercepting Mars in 120 days and returning to Earth in 258.5 days. In figure 7 results are shown for three and four corrections to intercept Mars with the last correction

being made at  $\frac{1}{10}$ ,  $\frac{1}{4}$ , and  $\frac{1}{2}$  days before intercept. Each of the other corrections were made after the vehicle traversed the fraction of the time to intercept shown on the abscissa. Each correction was assumed to have an error of 10 ft/sec along the orbit of Mars and 10 ft/sec along the radius to the Sun, or a total velocity error 14 ft/sec, which represents the combination of error in velocity, position, and thrusting. The guidance velocity shown is the total velocity for all corrections. Figure 8 shows the results for earth return with four and five corrections, with the last correction  $\frac{1}{10}$ - and  $\frac{1}{20}$ -day before perigee. After studying the results of figure 8 it was concluded that four or five corrections with the last correction being made at  $\frac{1}{10}$ -day or less were of most interest for the Earth return.

The results are presented as a function of the time of last correction, because as Sir Isaac Newton pointed out, position errors are a function of the integral of the velocity error after the last correction. Hence, the results presented for a last correction at 2 days before intercept represent about 20 times the error of a final correction at  $\frac{1}{10}$ -day. The error in a given vector direction of position, such as periapsis radius, however, depends on the direction of the velocity error. The error analysis of entering an aerodynamic corridor is not discussed here.

The results indicate that there is an optimum correction schedule and that the optimum schedule depends on the time of the last correction

and the number of corrections. Since the arrival errors are primarily a function of the error of the last correction and the time of the last correction, it is desirable to make the last correction as close to periapsis as possible. A typical schedule of guidance corrections for 120 days Mars transit time is shown in Table II for the  $\frac{8}{10}$  factor. The velocity error after each correction was the same as for figures 7 to 8, (14 ft/sec). This analysis does not account for either the rapid growth of error in the neighborhood of a planet or the improvement in navigation precision during the initial measurement period after injection into the trans-Mars (or trans-Earth) orbit. These effects indicate that a correction should be made as soon after launch vehicle burnout as a good estimate of state can be made. The next correction would not be made until 96 days after launch.

#### CONCLUDING REMARKS

The major conclusion to be drawn from this paper is that the Apollo navigation and guidance system is, in principal, adequate to perform the Mars mission. In fact, the efficiency of the Apollo system as it stands, except for details of the guidance, appears to be reasonable.

Summarizing the results presented, application of Apollo navigation techniques to the Mars Mission indicates that a 5-second RMS sextant will measure midcourse position and velocity accurately to about 300 nautical miles and less than 1/ft/sec. A schedule of one navigation sighting every 15 hours has been shown to be sufficient to maintain the estimate

of position and velocity at its minimum level. Position errors have been shown to be reduced to very low values in the last 10 hours of hyperbolic approach to periapsis of a planet. Velocity errors, however, tend to grow in this region, and a requirement for minimizing the time between the last navigation sighting and the last guidance correction is indicated.

A preliminary assessment of navigation and guidance techniques for Mars missions indicates that midcourse velocity requirements of 400 ft/sec for the trans-Mars phase shown in figure 7 and 600 ft/sec for the trans-Earth phase shown in figure 8 will be sufficient for injection and midcourse-correction velocity errors of 14 ft/sec or less. The correction schedule would require a minimum of four guidance corrections each way with the correction being made at  $\frac{8}{10}$  of the remaining time between last correction and perigee.

The major extensions of the Apollo techniques which evidently should be examined for application to the Mars mission are:

- (1) Consideration of the coupled navigation and guidance problem.
- (2) Use of a nonlinear navigation filter, a more exact technique than the linear filter used for Apollo.

- (3) Addition of estimation of geometric and physical quantities (such as the astronomical unit and gravitational properties of various bodies) to the filter.
- (4) Strict optimization of sighting and correction schedules.

#### REFERENCES

1. Battin, R. H.: "A Statistical Optimizing Navigation Procedure for Space Flight." MIT Instrumentation Laboratory Report R-341, revised May 1962.
2. Denham, W. F., and Speyer, J. L.: "Optimal Measurement and Velocity Correction Programs for Midcourse Guidance." Raytheon Report BR-2386, April 24, 1963.

Table I.- Space Navigation and Guidance System

Function	Equipment Type
I Navigation Measurements	A Sextant B Radar C LASER
II Orbit Determination (Position and Velocity)	Computer
III Arrival Error Prediction	Computer
IV Velocity Correction	Computer Attitude Reference Propulsion



TABLE II.- Typical Mars Guidance Correction Schedule  
Midcourse Navigation Error 14 ft/sec

Correction Number	Time of Correction		$\Delta V$ ft/sec
	From Earth, Days	From Mars, Days	
I Postinjection Correction	3	117	?
II First Midcourse Correction	96	24	96
III Second Midcourse Correction	115	5	71
IV Third Midcourse Correction	119	1	71
V Last Midcourse Correction	119.9	1	$\frac{142}{380+}$

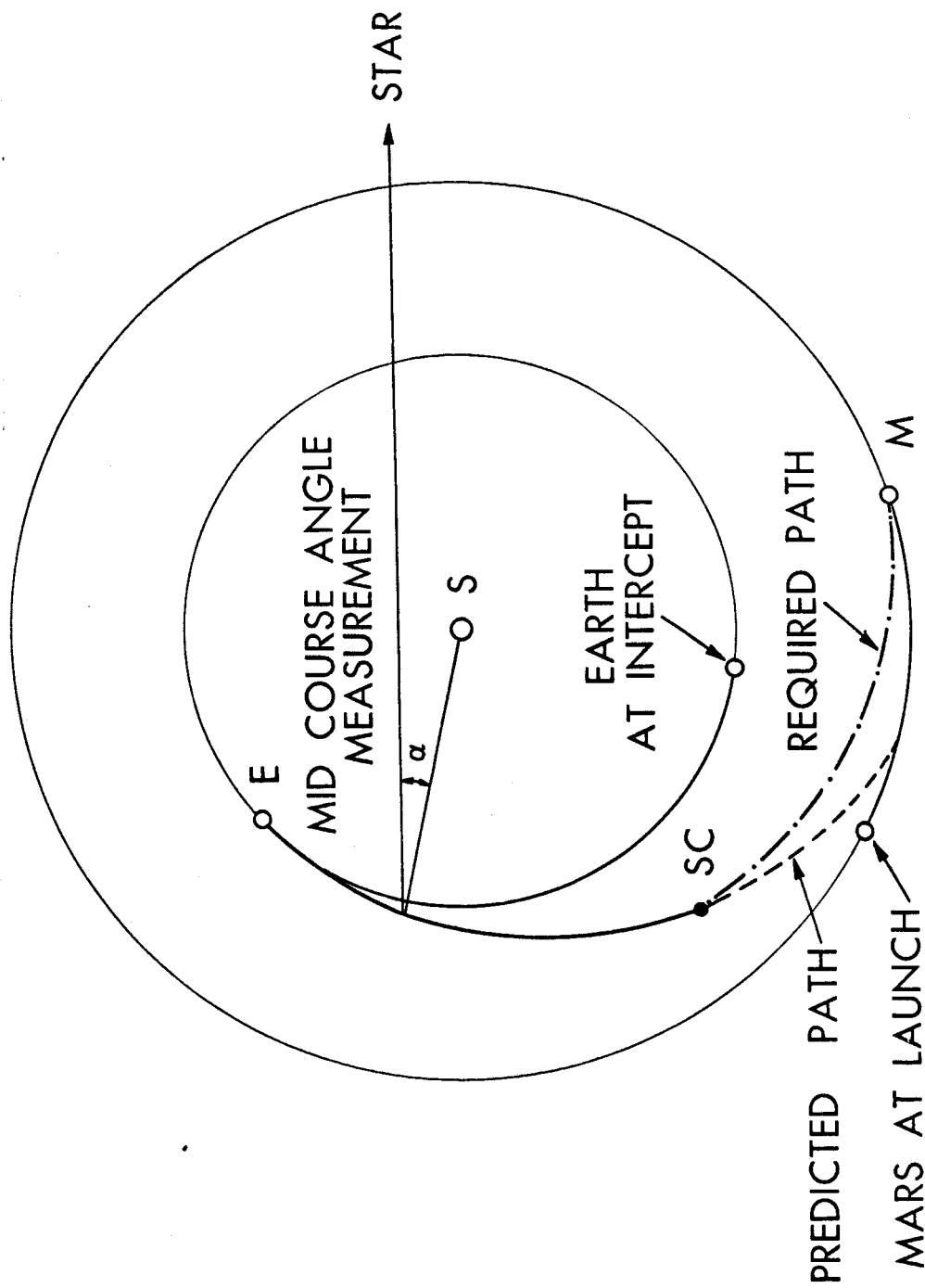


FIGURE 1. - SPACE NAVIGATION AND GUIDANCE MARS MISSION

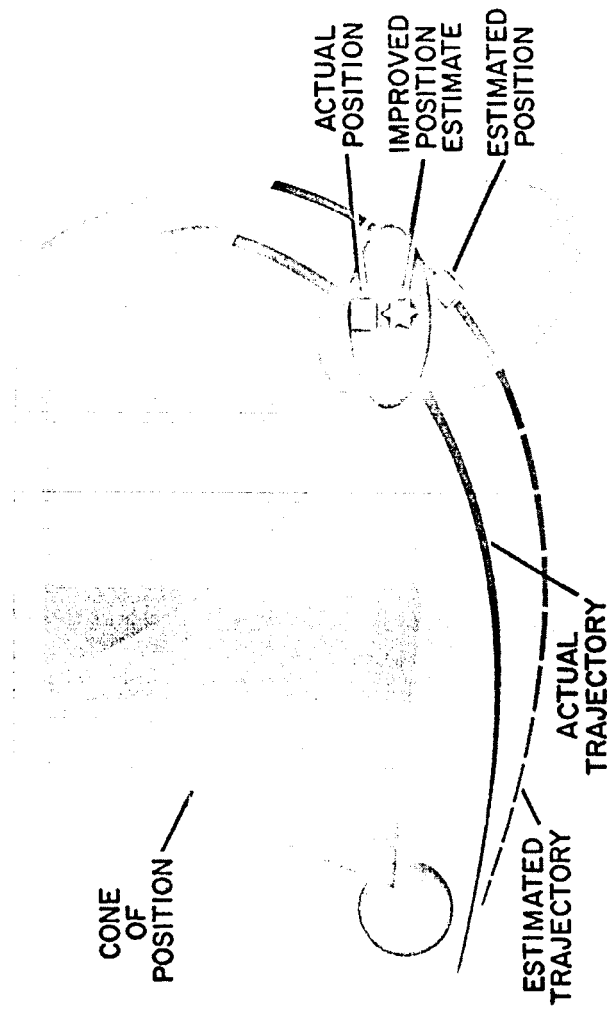


FIGURE 2. - TRAJECTORY ESTIMATION SCHEME

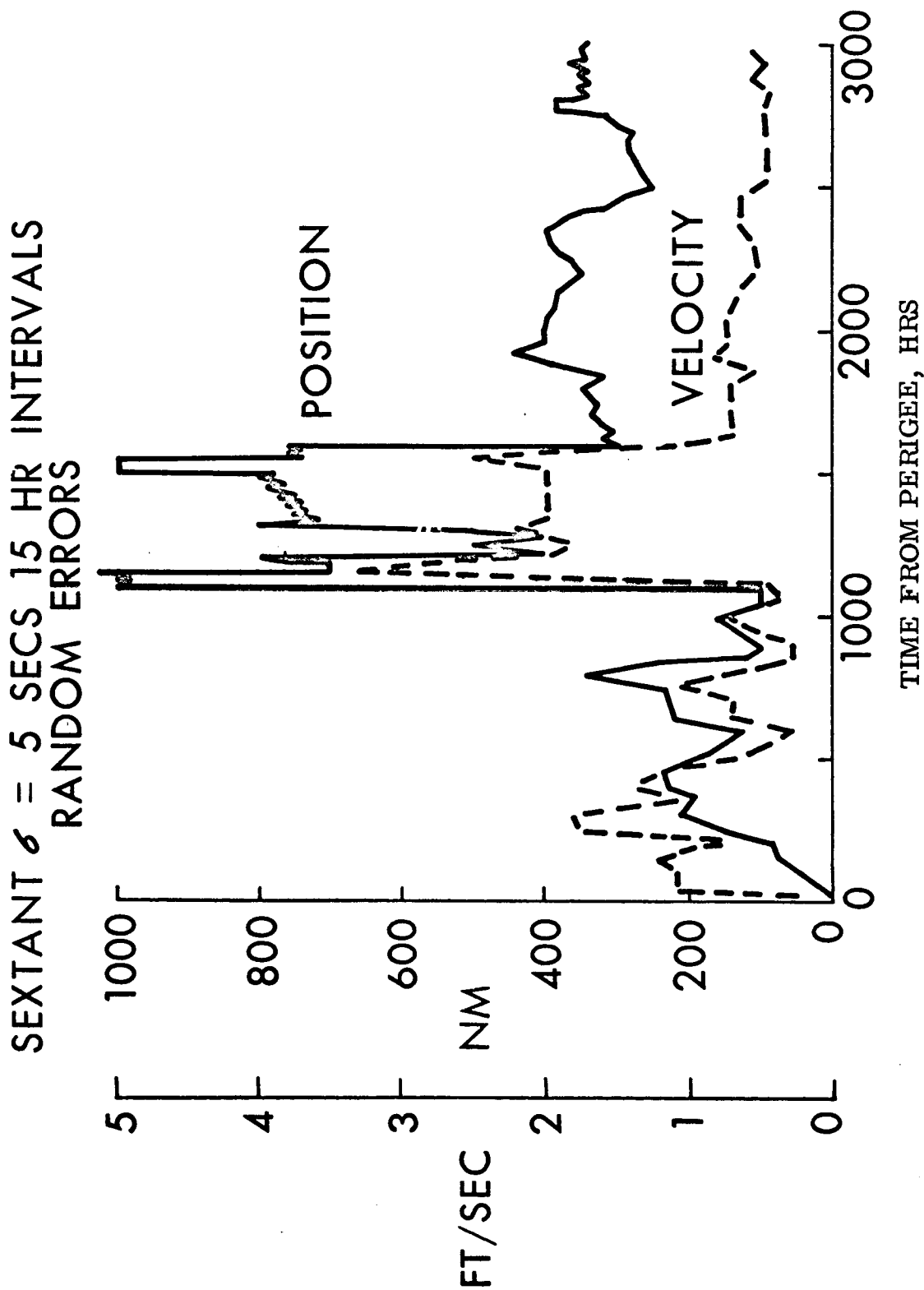


FIGURE 3. - MARS MISSION MIDCOURSE NAVIGATION ERRORS

SEXTANT  $\sigma = 5$  SECS      0.25 HR SIGHTINGS

RANDOM ERRORS

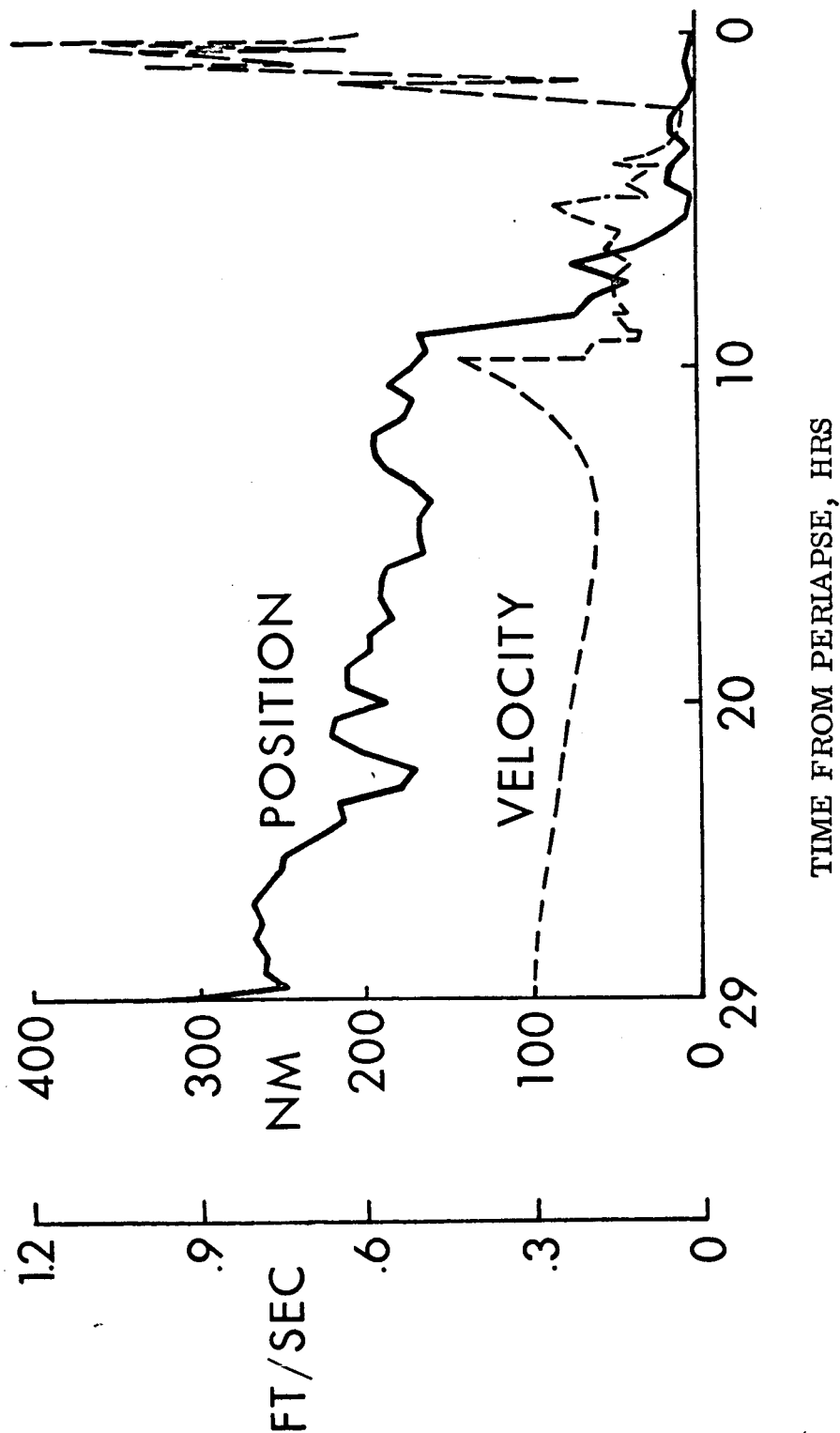


FIGURE 4. - MARS MISSION - MARS APPROACH NAVIGATION

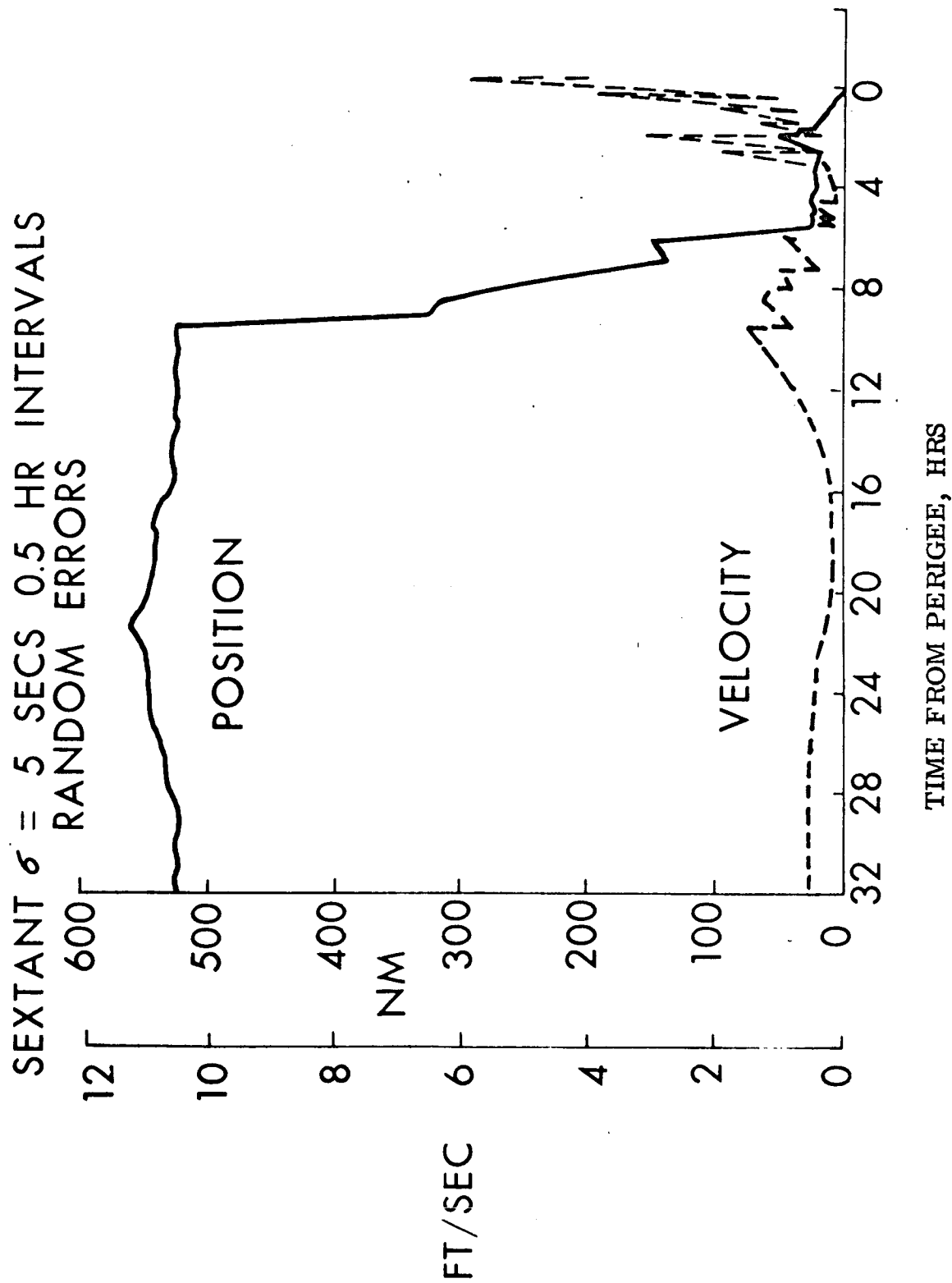
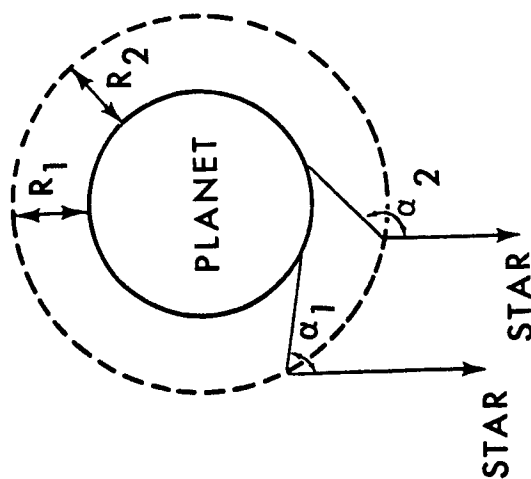


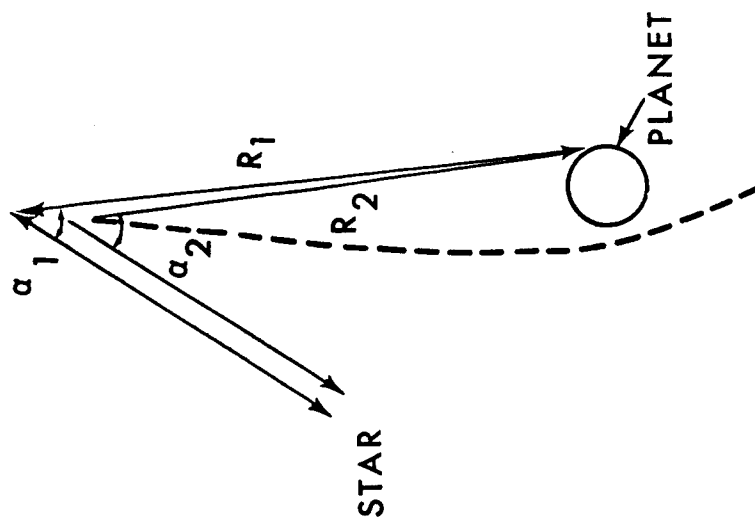
FIGURE 5. - MARS MISSION - EARTH APPROACH NAVIGATION ERRORS

# CIRCULAR ORBIT



ANGLE ( $\alpha$ ) IS A GOOD MEASUREMENT FOR DETERMINATION OF POSITION AND VELOCITY. RANGE (R) POOR

# HYPERBOLIC ORBIT



ANGLE ( $\alpha$ ) IS POOR MEASUREMENT FOR POSITION AND VELOCITY DETERMINATION UNTIL SEVERAL HOURS FROM PERIAPSIS. RANGE IS GOOD WHEN ANGLE IS POOR

FIGURE 6. - POSITION DETERMINATION NEAR PLANETS

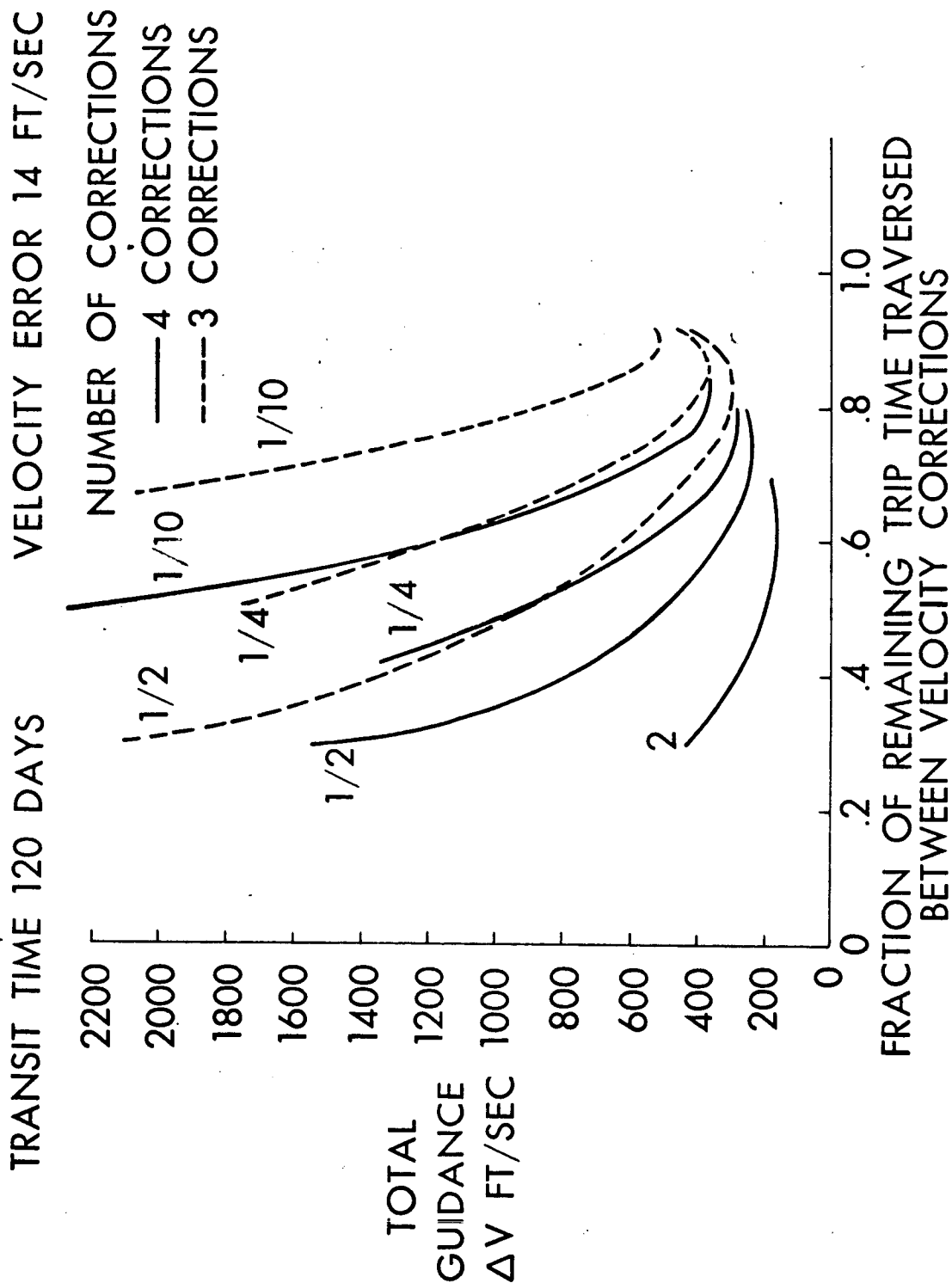


FIGURE 7. - MARS GUIDANCE VELOCITY REQUIREMENT



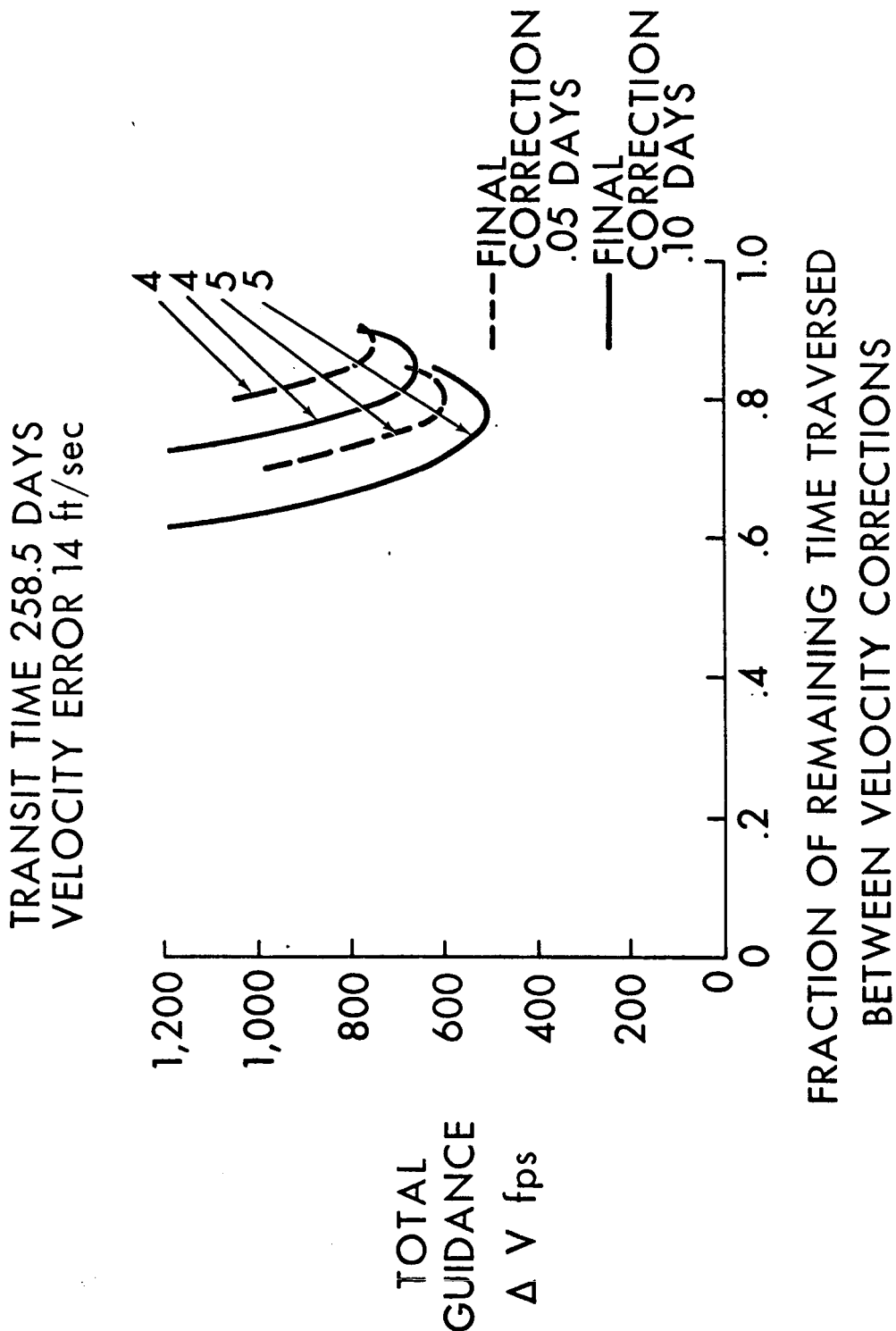


FIGURE 8. - MARS TO EARTH GUIDANCE VELOCITY